# A Famous Math Blunder

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NORTHERN ARIZONA UNIVERSITY
College of the Environment, Forestry, and Natural Sciences

I've watched a lot of TikToks recently about classic math mistakes. Some of them crack me up.

And I think/hope/pray that we just watched and listened to a couple of my favorite TikTok vids - this guy is super pissed about the two well known algebra errors that they ranted about...

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Here is one such story. It involves a famous problem in mathematical history about coloring maps, and is based on trying to answer this question...

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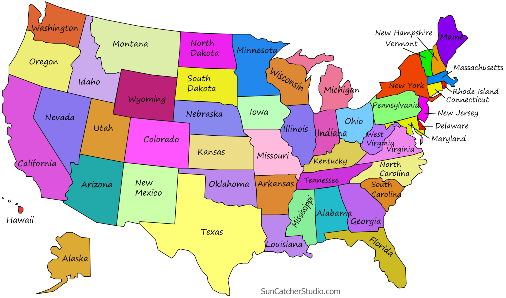


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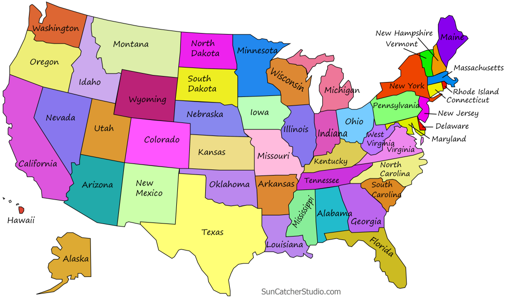
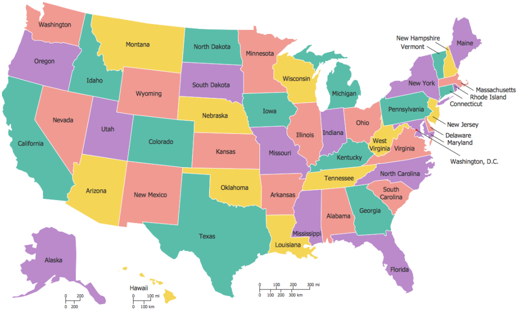



We see at least 9 different colors. Is 9 the *smallest* number of colors needed? No. Because I know a way more efficient way to color *'Murca*.

Here is a map of the United States colored with fewer colors:



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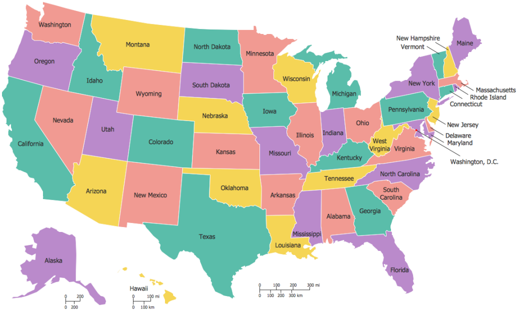


This time we see 4 different colors, which really *is* the minimal number of colors one needs to color a map of the United States. Look at the west coast for why we need more than 3 colors.

The real question in need of an answer is this:

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This is an old question, first posed by British mathematician Francis Guthrie in 1852. Guthrie believed that the correct answer is 4 colors, a claim that became known as the 4-Color Map Conjecture, and which remained unverified...until...

# Professional Mathematician Blunders, I(a)

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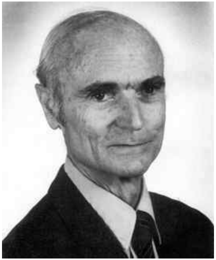

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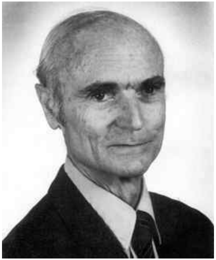

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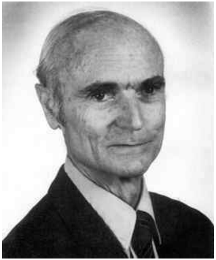

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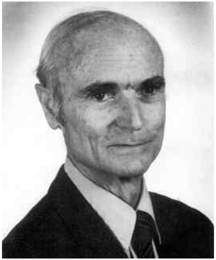

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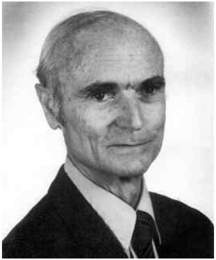

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Geez...Are mathematicians all dumb? No. They just make mistakes.

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The good news: there OFFICIALLY are no other mistakes in their proof.
The bad news: some mathematicians don't believe the proof is valid because it required computers to check for details. Sigh...

The math world rejoiced when the official and legit 4-Color Map Theorem was verified in 1989. It meant that people who make maps need no more than 4 colors in their fancy map books. But you know who doesn't care about all this map coloring stuff?

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*"Books on cartography and the history of mapmaking do not mention the four-color property."*

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# Möbius scarves!

# Grandmas care about the Möbius ?-Color Map Theorem, I

A Möbius strip is a one-sided surface. This one has ants on it, but lots of knitters and crocheters - Grandmas or otherwise - make colorful Möbius strip scarves!

# Grandmas care about the Möbius ?-Color Map Theorem, II

To make a colorful Möbius strip scarf, one must know how many colors are needed so that adjacent sections have different colors. You think I'm joking, but many people make Möbius strip scarves!

*Fiona Field* is one such Möbius artist. Here she is sporting both a Möbius scarf and a Möbius hat thingee. I think I see too many colors...

# Grandmas care about the Möbius ?-Color Map Theorem, III

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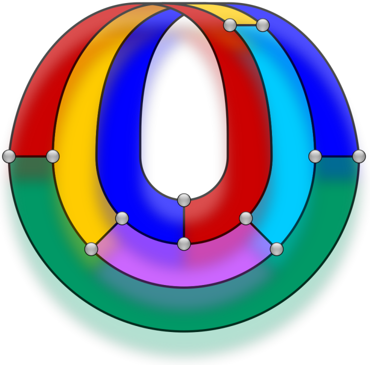

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What is the smallest number of colors one needs to color *any* map (or design) on a torus? [a bagel] It turns out that this is also a solved problem. The answer is...7.

The upper design only needed 5 colors, but the bottom design needed 7 colors, which is the minimal number needed for any design.

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# Questions?

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