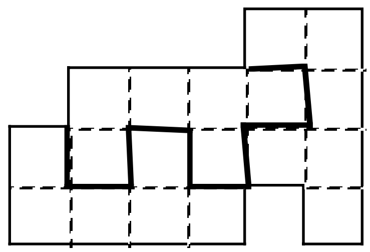
#1 SHOE

Draw lines to divide the shape below into 3 congruent shapes.



#2 FIRE WHEN GRIDDY, REDDLY!

						1 2 4
					2	2 3 6
						3 4 5
		1				1 2 6
			5			3 4 5
						1 5 6
2 4 5	1 3 6	1 4 6	2 3 5	1 3 4	2 5 6	

Fill in the square, one digit per cell so that:

(1) Each digit 1, 2, 3, 4, 5, 6 appears exactly 6 times

(2) Each row contains two each of the three digits listed to the right of it.

(3) Each column contains two each of the three digits listed below it.

Solution:Look at row 1 and column 2: They have only the number 1 in common, and so row 1 column 2 must be a 1. Similar thought gets you all of theleft board below

	1		2		2	1 2 4
2		6		3	2	2 3 6
	3	4			5	3 4 5
2		1	2	1		1 2 6
	3	4	5	5		3 4 5
5			5	1		1 5 6
2 4 5	1 3 6	1 4 6	2 3 5	1 3 4	2 5 6	

4	1	1	2	4	2	1 2 4
2	6	6	3	3	2	2 3 6
5	3	4	3	4	5	3 4 5
2	6	1	2	1	6	1 2 6
4	3	4	5	5	3	3 4 5
5	1	6	5	1	6	1 5 6
2 4 5	1 3 6	1 4 6	2 3 5	1 3 4	2 5 6	

Now look at row 5 and column 5. A 5 is there, so the 4's in that row must be in columns 1 and 3, while the 4's in column 5 must be in rows 1 and 3. Similar considerations fill in the board (above, right).

#3 NUMBER OF THE YEAR AWARD

All the counting numbers, beginning with 1, are written successively (that begins 12345678910111213....). What digit occupies the 2011th position?

Solution: the 1-digit numbers occupy the first 9 positions, leaving 2002. The 90 two-digit numbers occupy the next 180 positions, leaving 1822. The 900 three-digit numbers go out 2700 places, way past our goal. The first three-digit number, namely 100, uses up 3 places, leaving 1819. Dividing that by 3 gives 606 with a remainder of 1. So the last complete 3-digit number is (100 + 606) = 706. The last remaining digit is the first digit of 707, namely, 7.

#4 WEIGH IN

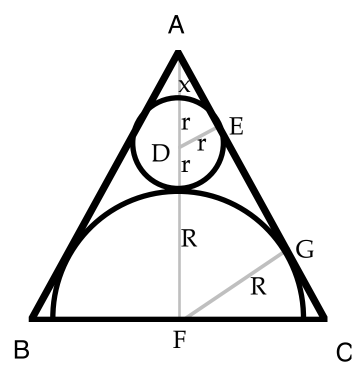
You are given 81 coins of the same denomination. You know that one of them is counterfeit and that it is lighter than the others. Show how to locate the counterfeit coin by using exactly four weighings on a pan balance.

Solution: A pan balance can give only 3 results: left heavier, right heavier, or both equal. Doing 4 weighings then can give 81 different results, just enough.
Group the coins into 3 stacks, A, B, C of 27 each, and weigh A against B. If A is lighter, the counterfeit is in A; if B is lighter,the counterfeit is in B; and if they are equal, the counterfeit is in C.

Whichever stack is indicated, divide it into 3 stacks of 9 each and weigh one against a second.. Continue for two more weighings to find the bad coin.

#5

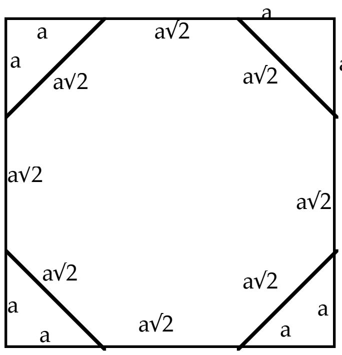
The figure shows an equilateral triangle ABC with an inscribed semicircle of radius R which is tangent to sides AB and AC, and an inscribed circle of radius r which is tangent to the triangle and the semicircle. Find the value of R/r



Solution: Draw altitude AF and radii DE and FG. Since ADE and AFG are 30-60-90 right triangles, each hypotenuse is twice the short leg. Thus r+x = 2r, and so r = x, and R+r+r+x = 2R, so R = r+r+x = 3r, and so R/r = 3.

#6 STOP, IN THE NAME OF MATH!

Bradley buys a square piece of Styrofoam to make a stop sign for a school musical. The Styrofoam costs four dollars. To make the stop sign, which is a regular octagon, Bradley cut off the corners of the square. What is the cost of the discarded portion of the Styrofoam?



Solution: If a is the leg of a discarded triangle, its hypotenuse is $a\sqrt{2}$, and so the area of the square is $(2a+a\sqrt{2})^2$. $= a^2(4+4\sqrt{2}+2) = a^2(6+4\sqrt{2})$. The area of the four triangles is $4(a^2/2) = 2a^2$. By propotionality, if C is the cost of the triangles, then $C/4 = 2a^2/(a^2(6+4\sqrt{2})) = 1/(3+2\sqrt{2}) =$ **$3-2\sqrt{2}$**.

#7 WHAT A DIFFERENCE A YEAR MAKES!

The sum of two numbers is 2011 and the difference of their squares is also 2011. What are the two numbers?

Solution: If 2011 is both A+B and A^2-B^2, $= (A+B)(A-B)$, then A-B = 1. Then 1+2011 = (A-B) + (A+B) = 2A, so A = **1006** and B = **1005**.

#8 MY BROTHER IS SUCH A LIAR!

Brad and Brett are identical twins. You have heard that one always lies, and the other always tells the truth, though you no one seems willing to tell you which is which.

You meet one of these two brothers at the hardware store, and, on impulse, you ask him,"Is Brett the liar?"

"No" he says.

What, if anything,, can you deduce from this?

Solution: You can't tell whether the person you are talking to is the liar or the truth teller, but you can be certain that he is . Consider: if he tells the truth, then Brett really *is not* the liar,and so your friend, who does not lie, is Brett. On the other hand, if you are talking to a liar, than Brett must be the liar, so again you must be talking to Brett.

#9 SOCKING

In a drawer are 10 identical blue socks and 10 identical red socks, all in a jumble. If you reach in and pull out two socks at random, what is the probability of getting a matching pair?

Solution: Pull out one sock and then a second. Whatever your first sock, there are 9 just like it among the remaining 19 socks. Your probability of picking one of those for your second sock is **9/19**.

#10 OUR GLASS

The points A, B, C, lie on the circle centered at O..AO is parallel to BC, and OB meets AC at D. Angle OAD is 24º. What is the measure of angle BDA?

Solution: By alternate interior angles, Angle DCB is 24º as well. Then DOA = BOA is twice that, or 48º. Then the exterior angle BDA is the sum of those, or 72º.