# 2011 Yavapai County Mathematics Contest <br> Individual Competition 

1. Simplify the expression $\frac{2^{4} 3^{2} 10^{2}}{6^{3} 5^{3}}$.
a) $\frac{8}{15}$
b) 1
c) 2
d) $\frac{3}{5}$
e) none of these
2. There are 100 cars on a used car lot. Out of those 100 cars, 35 are Toyota, 42 are Honda, and 45 are white cars. There are 18 white Toyota cars, and 20 white Honda cars. How many cars are not white, not Toyota, and not Honda?
a) 0
b) 7
c) 11
d) 16
e) 23
3. Certain types of engines require gasoline that is mixed with oil. How many liters of oil must be added to 5 liters of pure gasoline to get a mixture that is $20 \%$ oil?
a) 0.5 liters
b) 1 liter
c) $1 \frac{1}{4}$ liters
d) $1 \frac{1}{3}$ liters
e) $1 \frac{1}{2}$ liters
4. $A B=4, B E=5, D E=2$, and $m \angle A C B=m \angle D C E$. What is the length of $\overline{B C}$ ?

5. What is the value of $0.00025 \times 0.004$ ?
a) 0.00001
b) 0.000001
c) 0.001
d) 0.0000001
e) 0.00000001
6. Which is the line that contains the point $(2,-1)$ and is perpenducular to the line $7 x+5 y=4$ ?
a) $5 x+7 y=17$
b) $7 x-5 y=19$
c) $5 x-7 y=17$
d) $5 x-7 y=3$
e) $7 x+5 y=9$
7. What is the value of $\pi$ rounded to 3 decimal places?
a) 3.14
b) 3.140
c) 3.141
d) 3.142
e) 3.143
8. Which of the following statements are true for all real numbers $a, b, c$ ?
I $\sqrt{a^{2}+b^{2}}=a+b$
II $\quad \frac{a b+c}{a}=b+c$
III $(a+b)^{2}=a^{2}+b^{2}$
$\operatorname{IV} \sqrt{a^{2}}=a$
V $\frac{a+b}{a}=b$
a) I,III
b) IV
c) II,V
d) I,IV,V
e) none of these.
9. Simplify the expression $3 x y^{3}\left(2 x y-5 x^{2}\right)-8 x^{2} y^{2}\left(x y-3 y^{2}\right)$.
a) $-23 x^{3} y^{3}+30 x^{2} y^{4}$
b) $7 x^{3} y^{4}+24 x^{3} y^{4}$
c) $-23 x^{3} y^{3}-18 x^{2} y^{4}$
d) $9 x^{2} y+16 x^{3} y^{4}$
e) $6 x^{2} y^{4}+8 x^{3} y^{3}$
10. For what values of $y$ does the system of equations $\begin{gathered}2 x-9 y=22 \\ 4 x+3 y=2\end{gathered}$ have a solution?
a) $y=\frac{26}{9}$
b) $y=2$
c) $y=-2$
d) $y=\frac{-19}{6}$
e) no solution
11. Find the radius of the circle produced by the equation $x^{2}-2 x+y^{2}+12 y+33=0$.
a) 2
b) 4
c) 5
d) 10
e) 16
12. What is the solution set for the inequality $2 x^{2}+3 x>x^{2}+28$ ?
a) $\{x \mid x<-7$ and $x>4\}$
b) $\{x \mid x<-7$ or $x>4\}$
c) $\{x \mid-4<x<7\}$
d) $\{x \mid x>-7$ and $x>4\}$
e) $\{x \mid-7<x<4\}$
13. What is the largest solution to the equation $x^{4}-19 x^{2}+90=0$ ?
a) $x=-\sqrt{10}$
b) $x=3$
c) $x=\sqrt{10}$
d) $x=9$
e) $x=10$
14. Find all solutions to the equation. $|2 x-1|=x+5$
a) $x=6$
b) $x=4$
c) $x=4, x=6$
d) $x=\frac{-4}{3}, x=\frac{4}{3}$
e) $x=\frac{-4}{3}, x=6$
15. A furniture store will purchase a sofa from a manufacturer and then sell it to its customers for $150 \%$ more than what they paid for it. If the furniture store sells a sofa to a customer for $\$ 800$, how much profit do they make?
a) $\$ 200$
b) $\$ 266.67$
c) $\$ 320$
d) $\$ 480$
e) $\$ 533.33$
16. A rectangle has area 50 . When the length is decreased by 2 and the width is increased by 2 the new rectangle has area 58. What is the diffence in the length and width of the original rectangle.
a) 12
b) 2
c) 0
d) 8
e) 6
17. Compute the sum $1+3+5+7+9+\cdots+999$.
a) 249,500
b) 250,500
c) 250,000
d) 500,000
e) 499,000
18. Solve for $x . \log _{3}(2 x+1)=4$
a) $x=40$
b) $x=81$
c) $x=\frac{11}{2}$
d) $x=0$
e) $x=13$
19. What is the probability of getting 2 heads and 2 tails in 4 tosses of a fair coin.?
a) $\frac{1}{2}$
b) $\frac{3}{4}$
c) $\frac{3}{8}$
d) $100 \%$
e) $\frac{2}{3}$
20. Which inequality is true for all real numbers $a, b$ ?
a) $|a+b|<|a|+|b|$
b) $|a+b| \leq|a|+|b|$
c) $|a-b| \leq|a|-|b|$
d) $|a+b| \geq|a|+|b|$
e) $|a-b|>|a|-|b|$
21. The rectangle with vertices at ABCD is inscrbed inside a circle as shown. $B C=6, A B=2$.

What is the area of the C ircle?

a) $6 \pi$
b) $8 \pi$
c) $10 \pi$
d) $12 \pi$
e) none of these
22. Simplify the expression involving complex numbers. $(1+2 i)(3-2 i)+i(6-5 i)$
a) $10-12 i$
b) $12+10 i$
c) $4+14 i$
d) $8+13 i$
e) $-8+10 i$
23. How many arrangement of the digits 35916 produce prime numbers?
a) 0
b) 1
c) 2
d) 5
e) 10
24. What is the horizontal asymptote for the function $f(x)=\frac{(2 x-1)(4 x+7)}{(x+6)(5 x-9)}$
a) $y=\frac{4}{5}$
b) $y=2$
c) $y=8$
d) $x=-6, x=\frac{9}{5}$
e) none of these
25. Which is the eqation for the parabola for the given graph?
a) $y=(x-2)^{2}-7$
b) $y=-2(x-4)^{2}+7$
c) $y=2(x-2)^{2}-7$
d) $y=2(x+2)^{2}-7$
e) $y=(x-7)^{2}-2$

26. Find all solutions to the equation $\sqrt{2-x}=x-1$.
a) $x=\frac{1+\sqrt{5}}{2}$
b) $x=\frac{1 \pm \sqrt{5}}{2}$
c) $x=\frac{3 \pm \sqrt{13}}{2}$
d) $x=\frac{3+\sqrt{21}}{4}$
e) none of these
27. For any two numbers $x, y$ what is the value of $\frac{x+y+|x-y|}{2}$ ?
a) $x$
b) 0
c) $x+y$
d) maximum of $x, y$
e) minimum of $x, y$
28. For the function $f(x)=\frac{1}{x}$, find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $h \neq 0$.
a) $\frac{1}{h^{2}}$
b) $\frac{-1}{x^{2}+h x}$
c) 1
d) $\frac{1}{x^{2}+h}$
e) $\frac{-1}{x h}$
29. The points A and C lie on the larger of 2 concentric circles. The line segment $\overline{A C}$ is tangent to the inner circle at B . The area of the larger circle is 22 and the area of the smaller circle is 6 . What is the length of $\overline{A C}$ ?

a) $\frac{8}{\pi}$
d) $\frac{8}{\sqrt{\pi}}$
b) 2
e) $2 \sqrt{\pi}$
30. Grades in a class are determined by the formula $G=\frac{20 H+20 Q+40 E+20 F}{100}$. $G=$ grade in the class, $H=$ homework grade, $Q=q u i z$ grade, $E=$ exam grade, $F=$ final exam. Students receive an A in the class if their $G \geq 87.5$. Suppose that a student has the following scores going into the final exam: $H=90, Q=80, E=85$. What score must the student get on the final exam to get an A in the class?
a) 87.5
b) 92.5
c) 95
d) 97.5
e) 100
31. What is the missing number in the sequence $0,2,8,24,64$, $\qquad$ 384, 896, 2048,...?
a) 132
b) 160
c) 186
d) 212
e) 224
32. Each boy in a family has as many sisters as brothers, but each sister has only half as many sisters as brothers. How many siblings are in this family?
a) 5
b) 6
c) 9
d) 10
e) none of these.
33. Solve for $x$ in the equation $3^{x-2} \cdot 27^{x}=9^{x^{2}}$.
a) $x=\frac{-2}{3}$
b) $x=0$
c) $x=1, x=3$
d) $x=1$
e) $x=\frac{4}{3}$
34. If $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of the number is 5 , then what is 2 times $\frac{1}{8}$ of $\frac{4}{5}$ of the number?
a) 1
b) 4
c) 6
d) 8
e) 12
35. An open box is to be made from a 12 inch by 15 inch piece of cardboard by cutting out squares of length $x$ from the four corners and bending up the sides. Which of the following equations gives the volume of the box as a function of $x$ ?
a) $V=(12-x)(15-2 x) x$
b) $V=(12-x)(15-x) x$
c) $V=180 x$
d) $\quad V=180 x-4 x^{2}$
e) $V=(12-2 x)(15-2 x) x$
36. Suppose the operation $\square$ is defined on the set of integers as $a \llbracket b=a^{2}-2 b$.

Then what is the value of $3 ■(2 ■ 5)$ ?
a) -3
b) 15
c) 21
d) 28
e) 30
37. Which of the following does not have the same value as the others?
a) $\sqrt{7200}$
b) $10 \sqrt{72}$
c) $60 \sqrt{2}$
d) $4 \sqrt{450}$
e) $20 \sqrt{24}$
38. The equation $4 x^{2}-6 x+1=0$ has 2 solutions. What is their product?
a) $\frac{3}{8}$
b) $\frac{1}{4}$
c) $\frac{-1}{2}$
d) 2
e) none of these
39. In the figure below $2 \cdot A O=5 \cdot C O$. The area of the sector $D O C$ is $\frac{4}{5}$. What is the area of the sector $A O B$ ?

a) 2
d) 5
b) 4
e) 8.4
c) 4.2
40. The number $\pi$ is the ratio of what two values pertaining to a circle?
a) circumference to radius
b) circumference to diameter
c) area to radius
d) diameter to radius
e) area to circumference
41. Carlos can run half a mile in 4 minutes. What is his average speed in miles per hour?
a) 6 mph
b) 7.5 mph
c) 8.25 mph
d) 10 mph
e) 15 mph
42. What is the area enclosed by the figure below?

43. $f(x)=1-4 x, g(x)=x^{2}-3$. Compute $f(g(-2))$.
a) 78
b) 9
c) 11
d) -3
e) 29
44. There are 8 teams in a basketball division. Each team plays each of the other teams twice. How many games is this?
a) 28
b) 54
c) 64
d) 112
e) none of these
45. $2^{x}=7,7^{y}=11,11^{z}=16$ What is the value of $x y z$ ?
a) 1
b) 3
c) 4
d) 6
e) 8
46. Nine woodchucks can chuck eight pieces of wood in three hours. How much wood can a woodchuck chuck in one hour?
a) $8 / 27$
b) $8 / 3$
c) $1 / 6$
d) $3 / 8$
e) none of these
47. What is the $y$-intercept of the line that contains the points $(-3,7)$ and $(6,1)$ ?
a) 4
b) $\frac{15}{2}$
c) 5
d) 9
e) $\frac{9}{2}$
48. In the figure below $m \angle A D C=90^{\circ}, \overline{B D} \perp \overline{A C}, A B=8, B C=18$. What is the area of $\triangle A C D$ ?

a) 144
b) 156
c) 169
d) 132
e) none of these
49. Which is the graph of the equation $=3+4 \cos (\pi x)$ ?
a)

b)
c)


> d)


e)

50. Ronnie goes skiing at Snow Bowl. The line to get on the ski lift takes one third as long as the lift-ride up the mountain. The lift-ride up the mountain lasts for one half as long as it takes Ronnie to ski down to the bottom of the lift. Ronnie gets in the lift line at 10:00 am. He makes 4 complete runs up and down and then takes a lunch break at $12: 40 \mathrm{pm}$. How long does it take for Ronnie to ski down the mountain?
a) 15 min
b) 18 min
c) 20 min
d) 24 min
e) 30 min
51. What is the shape produced by the graph of the equation $|x|+|y|=10$ ?
a) circle
b) triangle
c) parabola
d) square
e) trapezoid
52. Given that $\tan \theta=-7$ and $\frac{\pi}{2}<\theta<\pi$, What is the value of $\csc \theta$ ?
a) $\frac{\sqrt{50}}{7}$
b) $\frac{1}{7}$
c) $-\sqrt{48}$
d) $-\sqrt{50}$
e) $\frac{-\sqrt{48}}{7}$
53. 1000 mg of a drug is given to a patient. The amount (in mg ) of the drug absorbed in the bloodstream $t$ hours after the drug has been taken is given by the function $D(t)=-40 t^{2}+400 t$. The drug is only effective when there is 360 mg or more absorbed in the blood. How long after the drug is taken does it stop being effective?
a) 1 hr
b) 5 hrs
c) 6 hrs
d) 9 hrs
e) 10 hrs
54. A circle is inscribed inside a triangle with sides of length 13,13 and 10 . What is the radius of the circle?

a) $\frac{10}{3}$
d) $\frac{13}{5}$
b) $\sqrt{\frac{23}{2}}$
e) none of these
55. The three points $(0,0)(3,4)$ and $(x, 1)$ are the vertices of an isoscoles triangle ( 2 sides having same length).

Which is not a possible value of $x$ ?
a) -1
b) $\sqrt{24}$
c) $\frac{17}{6}$
d) 7
e) $\frac{\sqrt{12}}{3}$
56. The points $\mathrm{A}, \mathrm{B}, \mathrm{D}$, and F are the vertices of a rectangle. Inside the rectangle is another rectangle with vertices at $\mathrm{C}, \mathrm{E}, \mathrm{G}$, and H where $\mathrm{C}, \mathrm{E}$ and G lie on the edges of the outer rectangle and H lies on a line connecting A to C . $A B=4, B C=2, C D=3$. What is the area of the triangle $\triangle A G H$ ?

a) $\frac{45}{8}$
b) $\frac{45}{16}$
c) $\frac{15}{4}$
d) $\frac{75}{16}$
e) $\frac{35}{8}$
57. If $\log _{a} b+\log _{b} a=5$, then what is the value of $\left(\log _{a} b\right)^{2}+\left(\log _{b} a\right)^{2}$ ?
a) 20
b) 23
c) 25
d) 27
e) 29
58. 3 circles in a line each with radius 1 are inscribed inside a semicircle as shown below. What is the radius of the semicircle?

a) $\sqrt{10}$
d) $2+\sqrt{3}$
b) $2 \sqrt{2}$
e) $1+\sqrt{5}$
c) $\sqrt{6}$
59. In an electrical circuit, the total resistance of two separate, parallel resistors can be calculated using the formula: $R_{T}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}$. Solve this equation for $R_{2}$.
a) $\quad R_{2}=\frac{R_{1}-R_{T}}{R_{1} R_{T}}$
b) $\quad R_{2}=\frac{R_{1}+R_{T}}{R_{1}-R_{T}}$
c) $\quad R_{2}=\frac{R_{1} R_{T}}{R_{1}-R_{T}}$
d) $\quad R_{2}=\frac{R_{1} R_{T}}{R_{1}+R_{T}}$
e) none of these
60. Which graph has a shaded region for the solution set of the inequality $36-9 x^{2}-4 y^{2} \geq 0$ ?
a)
b)
c)



d)
e)



